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AN EXPERIMENTAL STUDY OF CHOKED FOAM FLOWS IN A CONVERGENT–DIVERGENT NOZZLE

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Abstract—Choked flow of a foam in a convergent–divergent nozzle has been investigated. The foam consisted of air and a solution of a surface active agent in water. The upstream gas–liquid volume ratio δ_0 was in the range 0.053–1.57. The experimental results are in very good agreement with a homogeneous frictionless nozzle flow theory, assuming isothermal behaviour of the gas and no relative motion between the phases, for throat gas–liquid volume ratios δ_t as high as 0.8; for ratios in the range $0.8 < \delta_t < 2.98$ the agreement, while only approximate, is still quite close. Departures from the homogeneous theory are explained in terms of (a) the failure of the assumption of the isothermal behaviour and (b) the existence of relative velocity between the phases. The latter effect predominates at low values of δ_t but at large values, it appears that both contribute to errors in the predictions.

1. INTRODUCTION

Flow of two phase mixtures in a nozzle is a complex process and a problem of increasing technological importance. Nozzle flow occurs in control and throttling valves, metering equipment and inadvertently when a pressure vessel is punctured. The present work is an experimental study of foam flows in a convergent-divergent nozzle under an overall pressure ratio sufficient to choke the flow at the nozzle throat.

A vast amount of theoretical work has been developed to describe choking two phase flows and the propagation of acoustic waves in bubbly liquids. The simplest theory for describing gas–liquid flows is the "homogeneous-equilibrium model" (Wallis 1969) in which it is assumed that there is no mass transfer between the phases, the phases having the same velocity and temperature (gas expanding isothermally) and being in thermodynamic equilibrium everywhere along the nozzle. Thus, the mechanical behaviour of such a fluid is influenced mainly by the compressibility of the gas phase and the inertia of the liquid phase, assuming that heat, momentum or mass transfer between the phases proceeds to equilibrium.

The approximation of the homogeneous model can be only applied to a real flow if the two phases are well-mixed and it is certainly not true for slug, annular or stratified flows. Even gas–liquid bubble systems may depart from temperature and velocity equilibrium between the phases. Thus for equality of velocity, the gas bubbles should be very small and uniformly distributed so that they will be dragged along with the fluid at approximately the same velocity by the viscous forces ($\propto R^2$, where R is the radius of the bubbles) rather than be accelerated with respect to the liquid phases by the pressure forces ($\propto R^3$). Furthermore, due to the large heat capacity of the liquid phase and its good thermal contact with the gas-phase, the gas may also remain in thermal equilibrium with the liquid during expansion without appreciably lowering the liquid temperature. Under these conditions, the calculation of the velocity of sound through the mixture c, neglecting bubble response, is straightforward and yields (Wood 1941),

$$c^2 = \frac{P\delta}{\rho_L}\left(1 + \frac{1}{\delta}\right)^2 \qquad [1]$$

where δ is the volume ratio of gas to liquid, P, is the pressure and ρ_L the liquid density. A minimum in the sound velocity occurs at $\delta = 1$ and yields surprisingly low values of c. Thus at

$P = 1$ bar and $\delta = 1$, the sonic speed in an air water mixture is 20.1 m/s compared with 330 m/s in air and 1500 m/s in water, approximately.

Flow of two phase mixtures in a nozzle has been studied for the case of homogeneous, unidirectional flow by Tangren *et al.* (1949). They derived an analytical expression relating the critical pressure ratio, P_0/P_t (where P_0 is the stagnation upstream pressure and P_t the throat pressure) to the initial volume ratio of gas–liquid δ_0. At present, there is no adequate theory to describe non-equilibrium gas–liquid flows in a nozzle resulting in a simple useful expression as that of Tangren *et al.* (1949). A review by Hsu (1972) covers most of the work available.

Experimental investigations to test the above theories have been very scarce. The limited number of experiments of Tangren *et al.* (1949) and Muir & Eichhorn (1963) on air–water bubble mixtures, and the low volume ratio ($0.025 < \delta_0 < 0.12$) critical gas–liquid flows of Baum (1972) expanded in a nozzle of convergent angle 60°, show that these flows can be only approximately described by the homogeneous, unidirectional model and for $\delta < 1.0$. Work reported by Smith (1972), $15 < \delta < 675$, and Wallis & Sullivan (1972), on high volume ratio critical flows in an annular venturi, shows that the gas phase controls critical or near critical flows when the two phases are essentially in separate streams. A rise of $(P_0 - P_t)$ of only 10% was observed in moving from a mass ratio of gas–liquid μ of 0–1. The rest of the experimental work in nozzles has been directed towards an understanding of the structure of shock waves (Campbell & Pitcher 1958; Eddington 1970; Witte 1969; Muir & Eichhorn 1963) in the divergent section of the nozzle when sonic velocity is reached at the throat; the flow changing from supersonic to subsonic across the shock wave. Critical gas–liquid bubble flows in pipes have also been studied by Huey & Bryant (1967).

It appears that there is no systematic experimental work covering a wide range of volume ratios which can test the adequacy and limits of the homogeneous-equilibrium model, and very little work published on choked foam flows containing a surface active agent to impart foaming properties to the liquid phase.

The role of the detergent may be seen as aiding in the creation and maintenance of the two phases in equilibrium by reducing bubble coalescence and thereby maintaining a large proportion of very fine bubbles. Consequently, good thermal contact between the phases is maintained. The assumption of isothermal gas expansion may then still be valid at gas–liquid volume ratios above the limits for which a normal clean water bubble flow is no longer isothermal.

In this paper we present an experimental study of the flow of an air–water mixture containing a surface active agent, under choking conditions, through a convergent–divergent nozzle. In particular, we investigate the range of variables for which the homogeneous model is in good agreement with the data, and also the relative importance of relative velocity between the phases and non-isothermal behaviour in explaining deviations from the theory.

2. THEORETICAL ANALYSIS

2.1 *Basic assumptions for frictionless adiabatic foam flow*

Relationships between the critical pressure ratio P_0/P_t, and the choked mass flow rate, W as a function of the gas to liquid volume ratio at the nozzle inlet, δ_0 can be formulated with the following basic assumptions:

(a) The gas is in the form of bubbles which are uniformly distributed within the liquid phase and are of uniform size.

(b) The liquid phase is incompressible, i.e. $\rho_L \neq f(P)$ where ρ_L is the liquid density and P the pressure.

(c) The gas phase obeys the perfect gas laws, i.e. $\rho_G = P/RT_G$ where ρ_G is the gas density, and with constant specific heats C_p and C_v.

(d) There is no mass transfer between the phases, i.e. $\mu \neq f(P)$ where μ is the mass ratio of gas to liquid.

(e) Surface tension and vapour pressure effects are neglected, i.e. the pressures in the liquid and gas are equal.

(f) The gas and liquid are each assumed to have a uniform temperature, at any cross section of the nozzle.

(g) The flow in the nozzle is unidirectional, steady and frictionless.

Properties of the gas phase are indicated by subscript G and those of the liquid phase by L. Considering a unit volume of the foam, the mixture density, ρ is given by:

$$\rho = \frac{\rho_L}{1+\delta} + \frac{\rho_G \delta}{1+\delta}. \qquad [2]$$

The flow mass ratio of gas to liquid, μ, is independent of pressure and is given by:

$$\mu = \frac{\rho_G \delta k}{\rho_L} \qquad [3]$$

where k is the velocity ratio U_G/U_L.

2.2 *Equations of motion for frictionless adiabatic flow*

The equations for steady adiabatic frictionless flow are:
The mass conservation of liquid and gas requires that:

$$W_L = \frac{\rho_L U_L A}{1+\delta} = \text{constant}, \qquad [4]$$

$$W_G = \frac{\rho_G U_g A \delta}{1+\delta} = \text{constant}, \qquad [5]$$

where W is the mass flow rate and A is the nozzle area.

The momentum equation for the mixture is:

$$\frac{\delta}{1+\delta} \cdot \rho_G U_G \frac{\mathrm{d}U_G}{\mathrm{d}x} + \frac{1}{1+\delta} \rho_L U_L \frac{\mathrm{d}U_L}{\mathrm{d}x} = -\frac{\mathrm{d}P}{\mathrm{d}x} + \left(\frac{\rho_L}{1+\delta} + \frac{\rho_G \delta}{1+\delta}\right) g, \qquad [6]$$

where g is the gravitational acceleration.

Although viscous effects in the flow of the foam through a nozzle may be neglected, as a first approximation, the viscous drag on individual bubbles in the flow may be significant. The interphase drag force together with the virtual mass force, which is an inertial force arising from the relative motion between the bubbles and the surrounding liquid, cancel out when we add the momentum equation for each phase by applying Newton's third law. Consequently, the drag and virtual mass forces present in the momentum equation for each phase are equal but of opposite sign respectively.

However, there appears to be some controversy in the literature as to the omission of the virtual mass force in the mixture momentum equation. In an accelerating two phase flow, Prins (1974), Hinze (1969) and van Wijngaarden (1972) argue that not only the rate of change of relative velocity is influenced by the virtual mass but its effect may also result in an increase in pressure drop. In describing two phase bubble flows these authors have included the virtual mass term in the mixture momentum equation, i.e. the spatial rate of change of an impulse (relative motion between bubble and liquid × virtual mass of a bubble, $M = B\rho_L V_B U_G$ where B is the virtual mass coefficient and V_B the volume of a bubble).

Hinze (1969) who considered a control volume containing liquid and bubbles with the bubbles and liquid moving at different velocities and derived a momentum equation for the control volume which resulted in a virtual mass term. Consequently he concluded that the virtual mass term does not appear in the liquid momentum equation alone. He also claimed that Newton's law is not violated since the virtual mass force is also felt at the external boundaries of the liquid control volume as well as on the gas–liquid interphase, but with opposite sign. However, a strict formulation to show this effect was not given. This treatment differs from that given to the viscous drag force between the phases, which occurs in each of the momentum equations, but cancels on addition.

For a real two phase bubble mixture, it is virtually impossible to find an exact solution for the virtual mass coefficient, B. For a single bubble in an unbounded fluid with potential flow around the bubble $M = \frac{1}{2}\rho_L V_B U_G$ and $B = \frac{1}{2}$ (e.g. Lamb 1945). Thus, the virtual mass of a bubble flow takes the form $(\delta/(1+\delta))\rho_L U_G/2$ when the bubble intervals are sufficiently large that interference between the bubbles can be neglected. This is not generally true and the value of B has to be found from experiment (e.g. Prins 1974; Rose & Griffith 1965) although Zuber (1964) and van Wijngaarden (1976) have developed expressions relating the effective virtual mass of a bubble to the volume ratio in a gas-bubble/liquid mixture by considering the hydrodynamic interaction between bubbles on the liquid.

In most expositions of the equations of motion (e.g. Muir & Eichhorn 1963; Crespo 1969; Witte 1969), the virtual mass force is omitted from the mixture momentum equation and indeed Soo (1976) has observed that this force along with other interphase forces does not appear in this equation. The mixture momentum equation [6] is therefore, assumed to be physically acceptable.

If the gas velocity is related to the liquid velocity by $k = U_G/U_L$, and ρ_G is eliminated using [3], [6] becomes after rearranging:

$$\rho_L U_L \frac{dU_L}{dx}\left(\frac{1}{1+\delta} + \frac{\mu k}{1+\delta}\right) = -\frac{dP}{dx} + \left(\frac{1+(\mu/k)}{1+\delta}\right)g. \qquad [7]$$

The maximum velocity ratio in an accelerating bubbly flow is usually less than two (Wallis 1969) although much higher velocity ratios can occur in separated two phase flows (e.g. Fauske 1967; Levy 1964; Moody 1965).

The energy equation is:

$$\frac{\rho_G \delta}{1+\delta} U_G \frac{di_{TG}}{dx} + \frac{\rho_L}{1+\delta} U_L \frac{di_{TL}}{dx} = 0, \qquad [8]$$

where $i_T = i + U^2/2$ is the total enthalpy and where the two-dimensional kinetic energy associated with relative motion is assumed to be dissipated.

In addition, we also require an equation of state of the liquid phase,

$$\rho_L = \rho_L(T_L), \qquad [9]$$

where T_L is the liquid temperature, and also an equation specifying the expansion of the gas bubbles which is assumed to obey,

$$\frac{P}{\rho_G{}^n} = \text{constant}. \qquad [10]$$

Substituting ρ_G from [4] into [18] yields:

$$P(\delta k)^n = \text{const.} = P_0(\delta_0 k_0)^n. \qquad [11]$$

The volume ratio at any point along the nozzle is then given by:

$$\delta = \left(\frac{P_0}{P}\right)^{1/n} \cdot \frac{\delta_0 k_0}{k}. \qquad [12]$$

Although an equation specifying the heat transfer between the bubbles and the liquid is also required, estimates available in the literature for gas–liquid bubble flows (Tangren *et al.* 1949; Muir & Eichhorn 1963; Soo 1967) have shown that the flow can be considered isothermal at the initial temperature of the liquid. This would probably be the case in the present system of a fine-bubbled foam flow, the large heat capacity of the liquid phase and its good thermal contact with the gas phase maintaining the gas temperature constant. It thus appears that [8] and [9] can be dropped and [10], [11] and [12] replaced by setting $n = 1$, i.e.

$$\delta = \left(\frac{P_0}{P}\right) \cdot \frac{\delta_0 k_0}{k}. \qquad [13]$$

Substituting for δ from [13], the momentum equation [7] can be integrated from upstream initial conditions if the variation of the velocity ratio along the nozzle is known, i.e. $\partial k/\partial P$. The velocity ratio may also be a function of the volume ratio δ. An estimate of this value can be found experimentally if it is possible to distinguish visibly between the two phases in the convergent section of the nozzle. However, due to the high flow velocity and milky appearance of the flow in the present study, a detailed study of bubble velocities was not possible by high speed photography.

To simplify matters, the qualitative effect of the velocity ratio on the critical pressure ratio and choked flow rate can be shown by assuming (a) $k_0 = k = \text{constant}$ and (b) a simple arithmetic mean ($\bar{k}$) with no initial relative velocity so that $k_0 = 1$, i.e.

$$\bar{k} = \frac{k_0 + k_t}{2} = \frac{1 + k_t}{2}. \qquad [14]$$

Integrating [7] results for the liquid velocity U_L, when $k = \bar{k}$,

$$\rho_L \frac{U_L^2}{2P_0} = \rho_L \frac{U_0^2}{2P_0} + \frac{1}{1 + \mu\bar{k}}\left[1 - \frac{P}{P_0} - \frac{\delta_0}{\bar{k}} \ln \frac{P}{P_0}\right] + \frac{\rho_L(1 + \mu/\bar{k})}{P_0(1 + \mu\bar{k})} gX \qquad [15]$$

and when $k_0 = k$,

$$\rho_L \frac{U_L^2}{2P_0} = \rho_L \frac{U_0^2}{2P_0} + \frac{1}{1 + \mu\bar{k}}\left[1 - \frac{P}{P_0} - \frac{\delta_0}{\bar{k}} \ln \frac{P}{P_0}\right] + \frac{\rho_L(1 + \mu/\bar{k})}{P_0(1 - \mu\bar{k})} gX \qquad [16]$$

where X is the vertical height of the convergent section of the nozzle.

The last term on the R.H.S. of both these equations is the contribution to U_L due to the gas bubbles. Equations [15] and [16] reduce to the velocity obtained by the homogeneous flow theory when $k_0 = k = 1.0$, (Tangren *et al.* 1949), i.e. on neglecting the gravity term,

$$\rho_L \frac{U_L^2}{2P_0} = \rho_L \frac{U_0^2}{2P_0} + \frac{1}{1 + \mu}\left[1 - \frac{P}{P_0} - \delta_0 \ln \frac{P}{P_0}\right]. \qquad [17]$$

2.3 *Velocity of sound*

The calculation of the velocity of sound in the foam is a straightforward procedure when the two assumptions of isothermal gas behaviour and velocity equilibrium between the phases are included. Under these conditions, the sound velocity in a homogeneous medium, c is given by (Wood 1941):

$$c^2 = \left(\frac{1+\delta}{\delta}\right)^2 \frac{P\delta}{(1+\mu)\rho_L}. \qquad [18]$$

For the foams to be investigated $\mu \simeq 10^{-3}$ for $\delta < 3.0$. Therefore [18] becomes:

$$c^2 = \left[1 + \frac{1}{\delta}\right]^2 \frac{P\delta}{\rho_L}, \qquad [19]$$

indicating that a minimum in sound velocity exists for a volume ratio δ of 1.0, in which case, at a pressure of 1 bar a mixture of air and water has a sound velocity of about 20 m/s which is substantially lower than the speed of sound in either of the two phases separately.

A more complex expression for the sound velocity through a non-equilibrium two phase mixture when the two above assumptions are relaxed, i.e. $k > 1$ and $n > 1$, has been developed by Baum & Horn (1971) by considering a pressure wave transmitted in the continuous liquid phase. This is given by,

$$c_L{}^2 = \frac{nP\delta}{\rho_L}\left(\frac{1}{\delta} + \frac{1}{k}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right)\left(1 + \frac{1}{\delta}\right) \qquad [20]$$

where $\mathrm{d}U_G/\mathrm{d}U_L$ is the relative acceleration between the gas and liquid phase.

The equilibrium velocity given by [19] is a function only of pressure and the volume ratio, and is independent of frequency as shown by Hsieh & Plesset (1961). However, it has been observed by Karplus (1958) and Mercredy & Hamilton (1969) that the velocity of acoustic waves in a two phase bubble mixture increases with increasing frequency and Gregor & Rumpf (1975) have further shown that it is also a function of bubble size. This implies that [19] is only valid as the frequency of the sound wave approaches zero (slow compression or expansion), so that the time available for momentum and heat transfer is sufficiently long for these processes to proceed to equilibrium. The rate of approach to equilibrium is obviously enhanced when the bubble size diminishes. Thus, the speed of sound given by [19] is the minimum value which can be attained in the foam, for a given volume ratio and pressure.

The effect of frequency on sound velocity can be accounted for in the variation of $\mathrm{d}U_G/\mathrm{d}U_L$ (Gregor & Rumpf 1975) and of the polytropic gas expansion index n (Mercredy & Hamilton 1969) in [20], respectively. From conservation equations for a two phase system, including terms specifying non-equilibrium momentum and heat transfer, Mercredy & Hamilton (1969) obtained for the velocity of sound:

$$c^2 = \gamma c_{\mathrm{is}}^2(1 + \omega^2/\lambda^2)/(\gamma + \omega^2/\lambda^2) \qquad [21]$$

where c_{is} is the isothermal speed of sound in a bubble mixture given by [19], ω is the frequency of the sound wave and λ the inverse relaxation time for heat transfer from bubbles to the liquid which is inversely proportional to bubble size. In a polytropic flow, $c^2 = nc_{is}^2$, so evidently with [21],

$$n = \gamma(1 + w^2/\lambda^2)/(\gamma + w^2/\lambda^2). \qquad [22]$$

The limits to n are $n = 1.0$ (isothermal) when $\omega \to 0$ and $n = \gamma$ (adiabatic), i.e. the ratio of the specific heats, C_{pG}/C_{vG}, when $\omega \to \infty$.

When the flow becomes choked in the nozzle, the velocity of the mixture equals the speed of sound at the throat of the nozzle. Therefore, at the throat, substituting for δ from [12],

$$c_L^2 = \frac{nP_t}{\rho_L}\left(\frac{P_0}{P_t}\right)^{1/n}\frac{k_0\delta_0}{k_t}\left[\left(\frac{P_t}{P_0}\right)^{1/n}\frac{k_t}{k_0\delta_0}+\frac{1}{k_t}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right]\times\left[1+\left(\frac{P_t}{P_0}\right)^{1/n}\frac{k_t}{k_0\delta_0}\right]. \qquad [23]$$

When $n = 1$, $k = 1$ and $\mathrm{d}U_G/\mathrm{d}U_L = 1$, [23] reduces to [19], the homogeneous two phase velocity and when $n = 1$, expressing the velocity in a dimensionless form it becomes for $k_0 = 1$:

$$\rho_L\frac{c_L^2}{2P_0} = \frac{\delta_0}{2k_t}\left(\frac{P_t}{P_0}\frac{k_t}{\delta_0}+\frac{1}{k_t}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right)\left(1+\frac{P_t}{P_0}\frac{k_t}{\delta_0}\right) \qquad [24]$$

and for $k_0 = k_t$:

$$\rho_L\frac{c_L^2}{2P_0} = \frac{\delta_0}{2}\left(\frac{P_t}{P_0}\frac{1}{\delta_0}+\frac{1}{k_t}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right)\left(1+\frac{P_t}{P_0}\frac{1}{\delta_0}\right). \qquad [25]$$

The critical pressure ratio (P_t/P_0) for isothermal gas expansion can be found by letting $U_L = c_L$, i.e. equating [15] and [24] to obtain:

$$\frac{\delta_0}{2k_t}\left(\frac{P_t}{P_0}\frac{k_t}{\delta_0}+\frac{1}{k_t}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right)\left(1+\frac{P_t}{P_0}\frac{k_t}{\delta_0}\right) = \frac{1}{1+\mu\bar{k}}\left(1-\frac{P_t}{P_0}-\frac{\delta_0}{\bar{k}}\ln\frac{P_t}{P_0}\right)+\rho_L\frac{U_0^2}{2P_0}+\frac{\rho_L(1+\mu/\bar{k})}{P_0(1+\mu\bar{k})}gX \qquad [26]$$

which can be solved numerically to obtain the critical pressure ratio, P_0/P_t, as a function of δ_0, k_t and $\mathrm{d}U_G/\mathrm{d}U_L$. Similarly [16] can be used with [25] for the condition when $k_0 = k_t$, i.e. initial relative velocity is present. In the absence of relative velocity, [26] reduces to the expression obtained by Tangren *et al.* (1949) for homogeneous flow:

$$\frac{\delta_0}{2}\left(1+\frac{P_t}{P_0}\frac{1}{\delta_0}\right)^2 = \frac{1}{1+\mu}\left(1-\frac{P_t}{P_0}-\delta_0\ln\frac{P_t}{P_0}\right)+\rho_L\frac{U_0^2}{2P_0}+\frac{\rho_L}{P_0}gX. \qquad [27]$$

2.4 *Choked mass flow rate*

Combining [4] and [5], the choked mass flow rate W of the foam is:

$$W = \frac{\rho_L U_{L_t} A_t}{1+\delta_t}+\frac{\rho_{G_t} U_{G_t} A_t \delta_t}{1+\delta_t}. \qquad [28]$$

By the use of [12] and [3] for δ and ρ_G respectively [28] becomes:

$$W = \rho_L U_{L_t} A_t (1+\mu)\left[1+\left(\frac{P_0}{P_t}\right)^{1/n}\frac{k_0\delta_0}{k_t}\right]^{-1} \qquad [29]$$

where $U_{L_t}(= c_L)$ is given by [23]. Combining [29] and [23] and expressing the choked flow rate in a dimensionless form (W^*) gives:

$$W^* = \frac{W}{A_t}\left(\frac{1}{P_0\rho_L}\right)^{1/2} = \frac{1+\mu}{1+\left(\frac{P_0}{P_t}\right)^{1/n}\frac{k_0\delta_0}{k_t}}\cdot\left(\frac{P_0}{P_t}\right)^{1-n/2n}\cdot\left(\frac{\delta_0 n k_0}{k_t}\right)^{1/2}$$
$$\times\left[\left(\frac{P_t}{P_0}\right)^{1/n}\frac{k_t}{k_0\delta_0}+\frac{1}{k_t}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right]^{1/2}\left[1+\left(\frac{P_t}{P_0}\right)^{1/n}\frac{k_t}{k_0\delta_0}\right]^{1/2}. \qquad [30]$$

When $n = 1$, $k_0 = k_t = 1.0$, and $\mathrm{d}U_G/\mathrm{d}U_L = 1.0$, [30] reduces to the dimensionless choked flow rate expressed by the homogeneous theory. The contribution of the relative motion to W^* can be expressed by setting $n = 1$; and for the case of $k_0 = 1$:

$$W^* = \frac{1+\mu}{1+\dfrac{P_0}{P_t}\dfrac{\delta_0}{k_t}} \cdot \left(\frac{\delta_0}{k_t}\right)^{1/2} \left[\frac{P_t}{P_0}\frac{k_t}{\delta_0} + \frac{1}{k_t}\frac{\mathrm{d}U_G}{\mathrm{d}U_L}\right]^{1/2} \left[1 + \frac{P_t}{P_0}\frac{k_t}{\delta_0}\right]^{1/2}. \qquad [31]$$

Another extreme case with non-isothermal gas expansion and no relative velocity is described by:

$$W^* = \frac{1+\mu}{1+\left(\dfrac{P_0}{P_t}\right)^{1/n}\delta_0} \cdot \left(\frac{P_0}{P_t}\right)^{1-n/2n} (\delta_0 n)^{1/2} \left[1\left(\frac{P_t}{P_0}\right)^{1/n}\frac{1}{\delta_0}\right]. \qquad [32]$$

3. EXPERIMENTAL

3.1 *Experimental apparatus*

The apparatus used to study foam flow through a nozzle is shown schematically in figure 1. Metered water (and foaming agent) and air flows are introduced into the foam generating pump, then fed into a vertical convergent–divergent nozzle and finally discharged at ambient pressure into the feed tank, where the foam collapses. The water is drawn off and recycled.

The foaming agent used was Teepol, an anionic surfactant (Shell Chemicals Ltd.) which was added to the water initially to make a 0.05% solution by volume. The effect of varying the Teepol concentration was not investigated, 0.05% being adequate to form a stable foam. Air and water flow rates were measured by a series of calibrated rotamers ranging from 1.7 to $100 \times 10^{-5}\ \mathrm{m^3/s}$, the air flow rates being corrected for pressure and temperature. The accuracy with which the flow rates were calculated (including uncertainties in pressure, temperature, calibration and errors in scale readings) was estimated to be ±1.6% or less for the water and ±2.4% or less for the air.

The foam generator was a simple centrifugal pump into which the two phases were introduced by a mixing tee. The foam produced by the action of the impeller was extremely uniform in structure. The size of the bubbles produced was found to be independent of gas flow

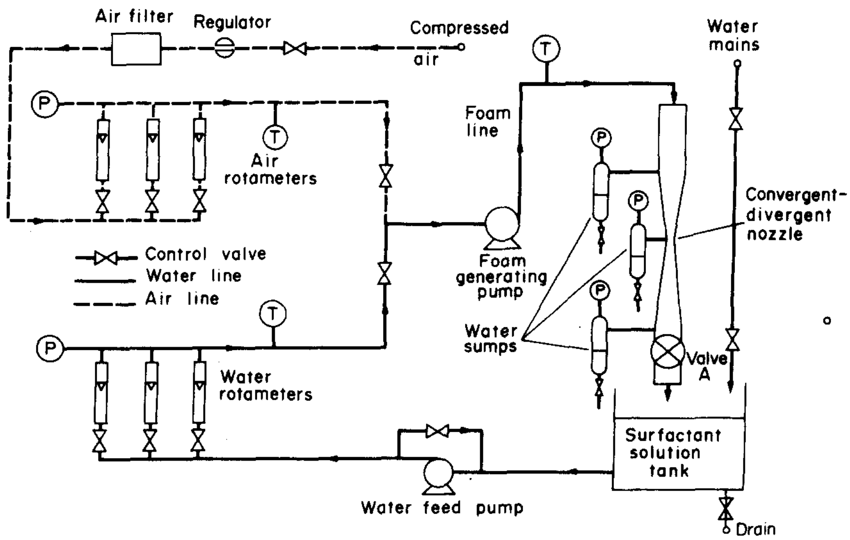

Figure 1. Schematic layout of apparatus.

rate. Starkey (1975) confirmed this observation by preparing histograms of bubble sizes taken from photographic prints and concluded that the mean diameter of the bubbles produced by the centrifugal pump generator was about 6×10^{-5} m. It is assumed that the air and water have the same velocity and temperature leaving the pump and hence the initial volume ratio (δ_0) can be directly calculated from the water and air flow rates.

From a knowledge of the flow characteristics of the water pump, and assuming a volume ratio of 1, preliminary calculations indicated that a throat diameter of 4 mm would reault in a throat velocity of over 20 m/s. Since the pressure is very sensitive to area in the vicinity of the throat, the point of minimum area must be located very accurately for the position of the pressure tap. This was found exactly by determining the area profile along the nozzle by means of a travelling microscope. An angle of 7° for the diverging section was considered to be the optimum value; losses due to possible separation and any two-dimensional effects being at a minimum at this value (Henry 1968).

Pressures along the nozzle were measured at the three relevant positions, namely (1) just upstream of the convergent section, (2) at the throat, and (3) at the end of the diverging section, by two 1.5 m mercury manometers for the throat and downstream tappings and by a calibrated Bourdon gauge for the upstream position. Since air was the transmitting fluid through the connecting tubes from the pressure taps to the measuring elements, sumps fitted with drain valves were installed at each of the pressure taps to prevent water being carried away into the lead lines under conditions of increasing pressure or due to seepage from the main flow. Pressure readings were accurate to within ±0.35 kN/m^2 except at values of $\delta_0>1.0$ when the accuracy was reduced to ±1.7 kN/m^2 due to larger pressure fluctuations.

A valve was installed downstream of the nozzle to control the nozzle exit pressure, P_D.

3.2 *Experimental procedure*

Before conducting experiments on the two phase mixture, flows with water alone at different flow rates were established in the nozzle and pressure measurements along the nozzle made. The purpose of these preliminary experiments was to assess the influence of wall friction. It has been shown by Wallis (1969) that wall friction in a turbulent bubble flow is solely due to the liquid phase—the gas phase having no significant effect. The results so obtained were translated into a pressure loss coefficient, K, defined as:

$$K=\frac{(P_0-P_t)}{\frac{1}{2}\rho_L(U_t^2-U_0^2)}. \qquad [33]$$

The values of K ranged from 1.03 to 1.0 for a range of water flow rates from 0.3 to 2.6×10^{-4} m^3/s, respectively. The pressure difference (P_0-P_t) was also calculated from the continuity and momentum equations for the liquid, neglecting wall friction; the measured and calculated pressure differences were within 3% indicating that the effect of wall friction was insignificant.

Basically three types of two phase flow experiments were carried out:

(1) A series in which increasing amounts of air were added to a constant volume flow of water, thus increasing δ_0, μ and P_0. The water flow rates were held constant at values of 1.0, 1.3, 1.7, 2×10^{-4} m^3/s for each series of runs.

(2) A series in which the upstream pressure P_0 was kept constant as the air flow rates were gradually increased (liquid flow rates subsequently decreasing).

(3) Runs in which air was introduced to the water flow at increasing flow rates without any restrictions on the upstream pressure or liquid flow rate.

A total of 95 runs were made during this study covering the following ranges: $0.053<\delta_0<1.57$; $2.40\times10^{-4}<\mu<65.0\times10^{-4}$; 113.5 *kN/m*$^2<P_0<445.0$ kN/m^2; 4.60 kN/m$^2<p_t<$

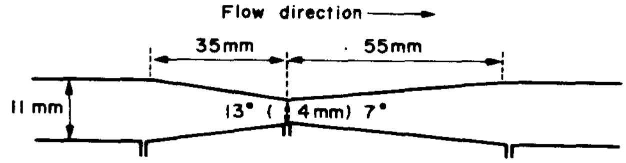

Figure 2. Details on convergent–divergent nozzle.

231.0 kN/m^2; 78.0 kN/m$^2 < p_0 <$ 109.5 kN/m^2; liquid flow rates Q_L, 1×10^{-4} m^3/s $< Q_L <$ 2.9×10^{-4} m^3/s, and gas flow rates at the throat Q_{G_t}, 0.5×10^{-4} m^3/s $< Q_{G_t} < 4.1 \times 10^{-4}$ m^3/s.

In all the runs, when the flow had been established and conditions were steady, the water and air flow rates, together with the pressures at the upstream, throat and downstream positions were read. The temperatures of the two phases entering the mixing pump and of the foam leaving the pump were also recorded.

3.3 *Visual observation*

Visual observation of the foam through the nozzle revealed several interesting points. The effect of mixing due to the centrifugal pump was clearly observed. Without the pump, the two phase mixture entering the nozzle was relatively transparent at low values of δ_0 and only relatively opaque at high δ_0. However, after passing through the pump, the foam had the appearance of an opaque white homogeneous fluid, it being impossible to distinguish between the two phases visibly, since the bubbles were about 50–100 μm in diameter. The foam appearance was generally a function of the gas–liquid volume ratio.

Shock waves, representing a discontinuity in the flow and indicating that sonic velocity had been reached at the throat, were distinctly observed in the diverging portion of the nozzle. Their position in relation to the throat, their size and oscillatory behaviour, also depended on the volume ratio. Shock waves only appeared when the throat pressure was less than or only slightly greater than the downstream pressure, which was approximately atmospheric in all the runs. This implied that for a constant upstream pressure of about 2 bars (near the minimum required for critical conditions), shock waves were only observed for $\delta_0 < 0.5$; the critical pressure ratio in the limiting case being about 2.0. The shock wave would tend to move towards or away from the throat as the air flow was lowered (decreasing δ_0) or raised (increasing δ_0), the wave appearing right at the end of the nozzle for $\delta_0 \simeq 0.5$ (see figure 3). The shock waves, appearing as black bands of thickness about 0.5 mm, covering the whole flow area and normal to the flow axis, were found to oscillate back and forth over 2–5 mm, the range of oscillation together with the size of the band itself increasing as δ_0 was increased. These runs were all steady.

There was a marked change in the appearance of the foam before and after the shock wave,

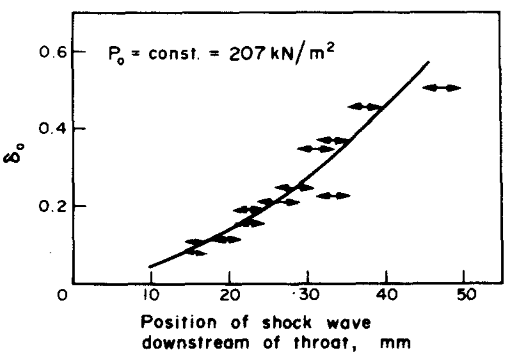

Fig. 3. Position of shock wave as a function of initial volume ratio: $T_L = 25$°C.

the foam being much more opaque due to bubble breakage after the shock wave as compared with the relatively transparent flow containing larger bubbles (due to expansion through the converging section) before the shock wave. This effect was more pronounced in the absence on the mixing pump, when the flow before the shock was almost similar to a jet flow. For runs were the initial volume ratio δ_0 was greater than 0.5, or alternatively, when δ_0 was less than 0.5 but the upstream pressure was much greater than the critical pressure ratio, shock waves were not observed in the diverging section, because the flow was wholly supersonic to the end of the nozzle. In these runs, the flow was opaque in the converging section but although clearer in the diverging section due to bubble growth and coalescence, it was still impossible to distinguish between the phases. Only in the extreme cases when the throat volume ratio, δ_t was as high as 1.7–3.0 were a cluster of bubbles of diameter approximately 1 mm observed near the exit of the nozzle, where δ was as high as 4–5. These high volume ratio flows became increasingly unsteady as δ_t approached values of 1.5 and above and a discontinuity in the flow in the connecting pipe outside the nozzle was observed. These "shock fronts" were much bigger than those in the low gas content runs and oscillated to and fro over distances as large as 10–20 mm, and moved further downstream as δ_0 was increased.

It has been observed by Huey (1967) for air–water bubble flows in pipes that when the volume ratio $\delta > 3.0$, the flow is no longer homogeneous, and the flow begins to slug. There was certainly no evidence of this in the foam flow. For δ as large as 5.5 at the nozzle exit, a certain degree of homogeneity still persisted in the foam.

3.4 *Criterion for a choked flow*

Although shock waves are an indication that the flow is sonic in the throat, this was confirmed in each run, whether sonic or sub-sonic, by steadily increasing the downstream pressure, P_D by closing valve A and taking readings of P_t, P_0, Q_G, Q_L and the position of the shock wave for each valve change.

When the flow was choked, these readings remained unaltered as the exit pressure increased, but the shock wave moved progressively upstream through the divergent section towards the throat (figure 4). This phenomenon is similar to that observed in single phase

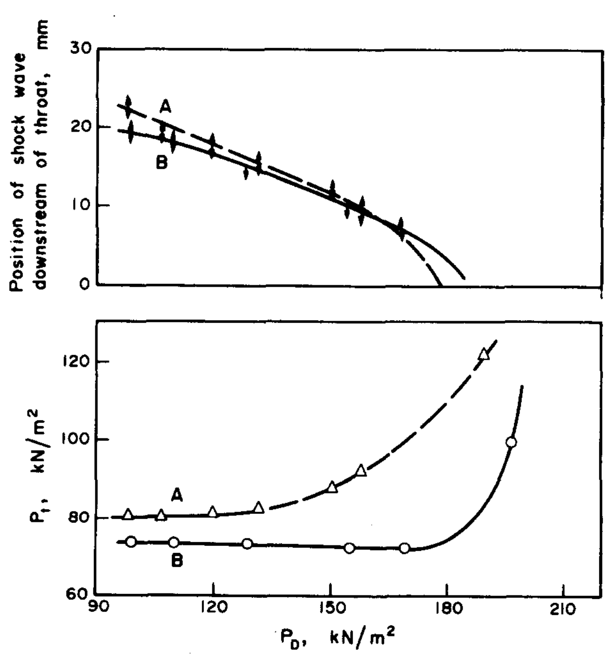

Figure 4. Typical profiles for confirmation of a choked flow for $\delta_0 = 0.19$ (Δ, A) and $\delta_0 = 0.12$ (o, B): $P_0 = \text{const.} \simeq 214\ \text{kN/m}^2$.

compressible flow. Abrupt changes in the throat and upstream pressures were only observed after the shock had reached the throat for conditions when the shock was very near the throat initially ($\delta_0 < 0.13$). However, for $0.13 < \delta_0 < 0.5$, although signals were just transmitted upstream by increasing the downstream pressure, the shock wave still persisted up to a distance of ~10 mm from the throat and only disappeared when P_D was further increased (figure 4). When $\delta_0 > 0.5$, the wave remained at the distance of about 20 mm from the throat. This behaviour has also been observed by Muir & Eichhorn (1963).

4. RESULTS

Results for the measured pressure ratio (P_0/P_t) and the dimensionless foam flow rate, W^* at the critical condition are presented in figures 5 and 6 respectively as a function of the initial volume ratio δ_0 and in figures 7 and 8 as a function of the throat volume ratio, δ_t which was calculated according to [12] assuming $n = 1$ and $k = 1$. Comparison is made with the values predicted by the homogeneous equilibrium theory ($k = 1$, $n = 1$, and $dU_G/dU_L = 1$) for two phase gas–liquid frictionless nozzle flow. The theoretical pressure ratios are calculated from [27] and the theoretical dimensionless flow rates from [32] using the measured pressure ratios and setting $n = 1$.

Both sets of graphs are important in that those with δ_0 as the abcissa serve as a design criterion to predict the critical pressure ratio and maximum choked flow rate for a mixture of known initial volume as in the case of the design of pressure relieving ducts. Graphs with δ_t as the abscissa scale are more relevant to this study in that they give an indication of the maximum range of δ for which the foam flow can be approximated to the homogeneous flow theory.

The results indicate that for $\delta_0 < 0.4$ (or $\delta_t < 0.8$) the measured pressure ratios and flow rates are in very good agreement with the equilibrium theory, the calculated and measured pressure

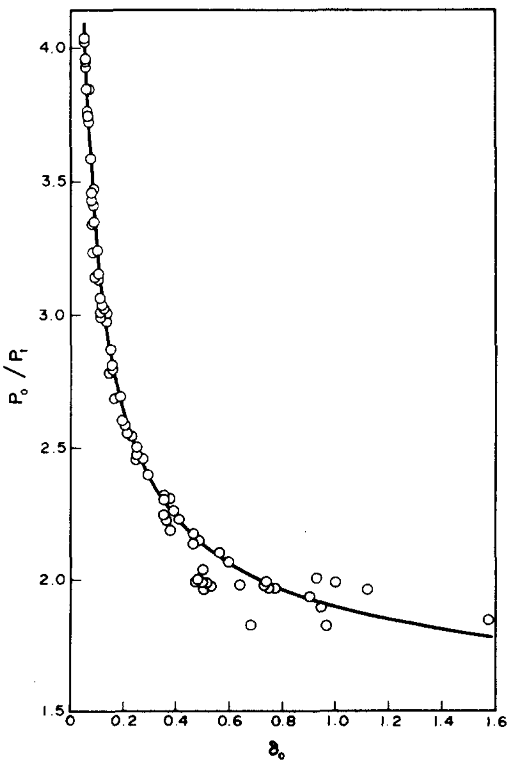

Figure 5. Comparison of experimental and theoretical (homogeneous-equilibrium theory) critical pressure ratio as a function of initial volume ratio: ——, theory [27]; O, measurement; $T_L = 25°C$.

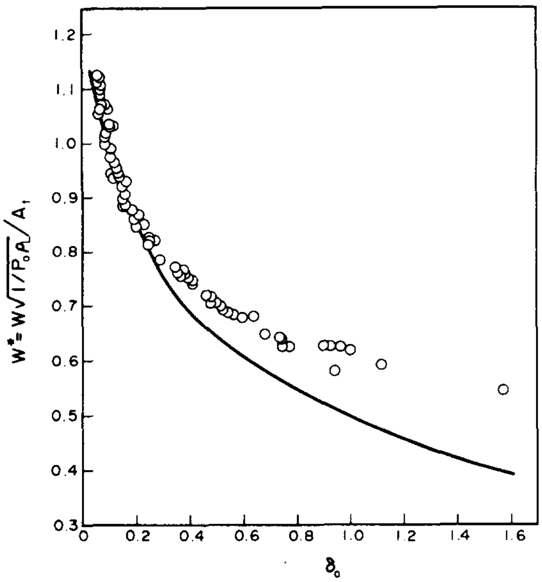

Figure 6. Comparison of experimental and theoretical (equilibrium model) dimensionless choked flow rate as a function of initial volume ratio: ——, theory [32] with $n = 1$; O, measurement; $T_L = 25°C$.

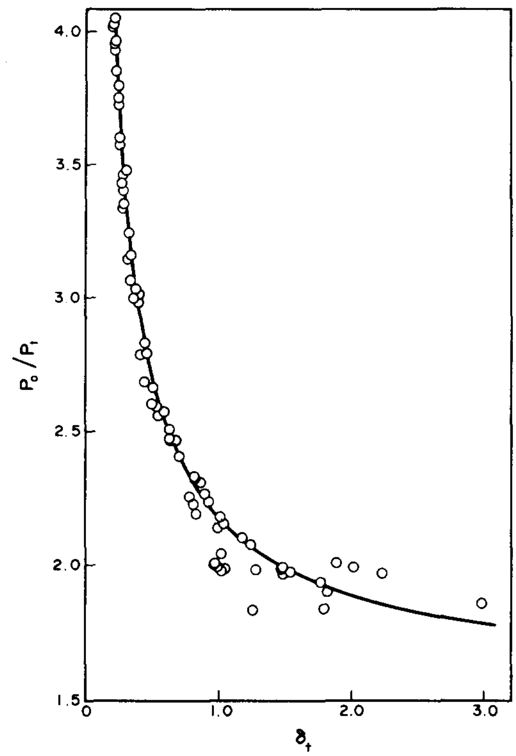

Figure 7. Comparison of the effect of experimental and theoretical (homogeneous theory) choked pressure ratio on throat volume ratio: ——, theory [27]; O, measurement; $T_L = 25°C$.

ratios being within 2% or less and the flow rates being with 7% or less. For $0.4 < \delta_0 < 1.57$ $(0.8 < \delta_t < 2.98)$ discrepancies occur. The pressure ratios are lower in the range $0.4 < \delta_0 < 0.7$ and higher for $\delta_0 > 0.7$, than the theory predicts while the flow rates are higher over the whole range. Nevertheless, the homogeneous isothermal flow model is still a very good approximation. Deviations are within 10% or less for the pressure ratios and within 25% or less for the choked flow rates within this range of δ.

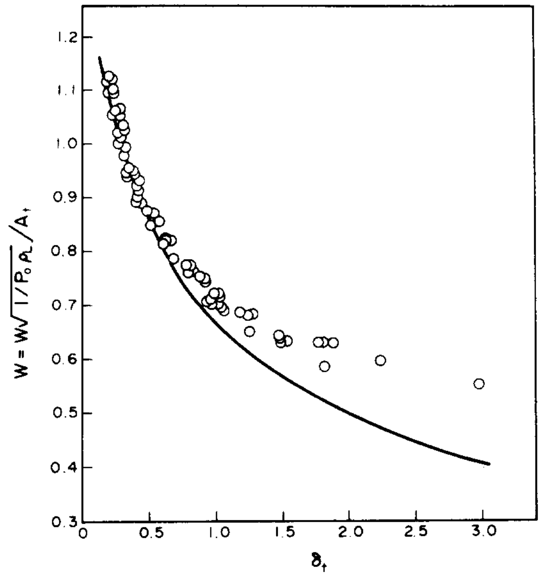

Figure 8. Comparison of the effect of experimental and theoretical (equilibrium model) dimensionless choked flowrate on throat volume ratio: ——, theory [32], with $n = 1$; ○, measurement; $T_L = 25°C$.

Velocities at the throat were calculated from the measured mass flow rates, W_{exp} and throat volume ratios, δ_t from [12] assuming $U_G = U_L$ and $n = 1$, i.e.

$$U_m = \frac{W_{exp}}{A_t}\frac{(1+\delta_t)}{\rho_L} \qquad [34]$$

and compared with the equilibrium speed of sound c through the foam [19]. U_m is actually the mean mixture velocity and is a physical entity only if $U_G = U_L$ when it becomes the same as the bulk mixture velocity as expressed by [17]. However, it is recommended by its ease of calculation and provides a useful basis for comparison. A truer representation would be to compare the theoretical homogeneous mixture velocity [19] with a measured liquid velocity, if this were possible, since the former may be equated to the liquid velocity. As mentioned earlier, this was not possible due to difficulty in distinguishing between the two phases.

Using the mean mixture velocity U_m, the Mach Number M defined as:

$$M = U_m/c \qquad [35]$$

ranged from 0.663 to 1.35 for the whole range of experiments. For the critical runs the values of M were $0.98 < M < 1.09$ for $\delta_t < 1.0$ and $1.09 < M < 1.35$ for $1.0 < \delta_t < 2.98$. Figure 9 confirms this and shows a similar trend to that observed in the critical pressure ratio and mass flow rate plots in that U_m becomes increasingly higher than the uniform velocity, isothermal speed of sound as δ_t increases above 0.8. The most interesting feature of figure 9 is that the measured values show a minimum in velocity at $\delta_t \simeq 1.0$, as indicated by the theory.

5. DISCUSSION

The results show that the homogeneous flow theory can be used adequately to describe critical foam flows for air–water volume ratios (in the throat) as high as 3.0. In comparison, where a surfactant is not used and a true foam flow is not achieved, the homogeneous theory is known to fail. Thus the experimental data of Baum (1972) on low-volume-ratio bubble flows ($0.025 < \delta_0 < 0.08$) show that the measured critical pressure ratios P_0/P_t are above the values predicted by the homogeneous theory, while for $0.08 < \delta_0 < 0.12$ they are below. The relatively

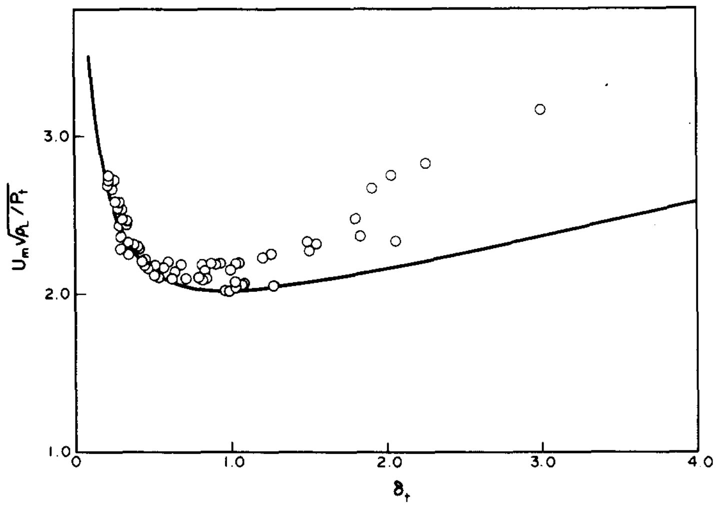

Figure 9. Experimental and theoretical dimensionless choking velocity as a function of throat volume ratio: ——, theory [19]; ○, measurement [34]; $T_L = 25°C$.

few data of Tangren (1949) also reveal that an air–water bubble flow is only approximately described by the homogeneous model for $\delta_t < 1.0$.

If the assumptions on which the homogeneous theory is based are examined in relation to the properties of the foam stabilized by a surfactant, it is not surprising that the foam flow corresponds so closely to the ideal homogeneous fluid. The bubbles generated in the foam are very small, of order 100 μm, and they are prevented from coalescing, at least in the time scale of the experiments, by the surfactant. Because of their small size, we would expect the viscous drag on the bubbles to be large compared with the acceleration effect which tends to cause relative velocity, so relative velocity would not be expected to be significant unless the bubbles became large or the liquid ceased to act as a continuous phase. The assumption of isothermal volume change is also reasonable with very small bubbles, because the liquid provides a large mass reservoir which can absorb the small heat changes in the gas with only an infinitesimal temperature change. Also, the bubbles are sufficiently small for the heat of compression to be conducted rapidly to the surroundings.

Although the results are in good agreement, it is clear that deviations from the homogeneous theory do occur at large gas–liquid volume ratios, possibly due to violation of these assumptions. Before considering these in detail we shall first consider two other possible reasons for these departures: mass transfer between the bubbles and water, and the effect of frequency on the speed of sound.

Rates of mass transfer between the phases can be estimated by considering the time necessary for a bubble to dissolve. This is of order R_m^2/D_G where D_G is the diffusion coefficient of air in water ($\sim 2 \times 10^{-9}$ m^2/s for N_2 or O_2). Thus for a bubble radius R_m of 10^{-4} m, R_m^2/D_G is 5s. The time required for the foam, moving with velocity U_m, to pass through the nozzle of effective length L, is L/U_m. Setting $L = 35$ mm and $U_m = 2$ m/s (an underestimate), we find L/U_m to be 0.017 s, which is much smaller than the bubble diffusion time. Accordingly, there will be negligible mass transfer between phases in passing through the nozzle.

It was indicated in §2.3 that the speed of sound in an equilibrium two phase bubbles mixture given by [19], is only valid when the gas bubbles are so small that the wavelength of the sound wave is large compared with the bubble size. This implies that the frequency must approach zero (Karplus 1958; Mercredy & Hamilton 1969) so that in a slow compression or expansion, the time available for momentum and heat transfer between the phases is sufficiently long for these processes to proceed to equilibrium.

In passing through the nozzle the foam undergoes a pressure change in the time scale of order ω^{-1}, ω being the "associated" frequency of the foam. To calculate ω we assumed that the pressure changes in the nozzle occur mainly over a length scale of about 5 mm of the nozzle throat, as was found by Baum (1972) in a geometrically similar nozzle, and this length is analogous to half a wavelength of an expansion wave. Thus the "associated" frequency for a foam velocity of $U_m = 17.9\ m/s$ (when $\delta_0 = 0.12$) is $2\pi\ (17.9 \times 0.5/5 \times 10^{-3})$ or 1.12×10^4 rad/s. At the highest volume ratio, i.e. when $\delta_0 = 1.57$, $U_m = 42.6$ m/s so that $\omega = 2.7 \times 10^4$ rad/s.

However, even at these high frequencies the gas may still expand isothermally provided the inverse relaxation time for heat transfer between the bubbles and the liquid, λ, is still large compared with ω, i.e. $\omega^2/\lambda^2 \ll 1$ so that $n \to 1$ in [22]. An estimate of λ can be found by considering the heat transfer by conduction to spherical bubbles from the surrounding liquid when subject to an oscillatory pressure field. Plesset & Hsieh (1960), assuming a homogeneous distribution of bubbles in the liquid showed that λ is given by:

$$\lambda = \frac{\rho_L C_L}{\rho_G C_{pG}} \frac{D_L}{R_m^2} \qquad [36]$$

where D_L is the liquid thermal diffusivity.

The above relation shows that $\lambda \to \infty$ if $\rho_L C_L \gg \rho_G C_{pG}$ (i.e. if the thermal capacity of the liquid phase is much larger than that of the gas) and if the bubbles are so small ($R_m \to 0$) that the heat of compression is rapidly conducted to the surrounding liquid. In the present foam we have $C_L/C_{pG} = 5.8$, ρ_L/ρ_G in the range $3.6 \times 10^2 - 1.8 \times 10^3$ (the gas densities evaluated at $P = P_t$) and R_m of order 50 μm. Thus λ, varying between $12.5\text{–}62.0 \times 10^4\ \mathrm{s}^{-1}$, is much greater than the "associated" frequencies. The condition of isothermal gas expansion, therefore, appears to be satisfied for the whole range of initial volume ratio used in the foam. However, the mean bubble radius would probably be larger than 50 μm at the higher range of volume ratio and would increase with increasing δ_0 with a subsequent reduction in λ; n may then be greater than unity according to [22].

Another possible reason for departures from the homogeneous theory is the effect of bubble resonance. Bubble resonance effects become important in sound transmission at high frequencies when the "associated" frequency is comparable with the bubble resonant frequency which is given by (Eller 1970):

$$\omega_R = \frac{1}{R_m}\left(\frac{3P}{\rho_L}\right)^{1/2}. \qquad [37]$$

The bubble resonant frequency calculated assuming $P = P_t$ was always large compared with the "associated" frequency even when a bubble radius as high as 300μm (an overestimate) was assumed, e.g. when $\delta_0 = 0.12$ and $R_m = 50\mu$m, $\omega_R = 27 \times 10^4$ rad/s and if $R_m = 300\,\mu$m, $\omega_R = 4.5 \times 10^4$ rad/s.

The bubbles observed in the experiments of Baum (1972) were of order 5 mm in radius, and calculations similar to the above show that in his case, $\omega > \omega_R$. This may be one reason why agreement with the homogeneous theory was not very good.

The effect of slip or relative motion between phases has already been considered by Muir & Eichhorn (1963). Their experimental results for an air–water choked bubble flow in a convergent–divergent nozzle indicated that the liquid velocities and choked flow rates were lower than predicted by the homogeneous theory for $0.1 < \delta_t < 3.0$. The corresponding critical pressure ratios P_0/P_T were also lower than predicted. They introduced the gas and liquid separately, without additional mixing and it is reasonable to presume that slip existed even before the mixture entered the nozzle. It can easily be shown that the homogeneous theory with allowance for slip is consistent with their observations, by solving [16] and [25] simultaneously and finding P_0/P_t numerically.

In our case, the foam is well mixed upstream of the nozzle and it is likely that slip does not exist at the entrance to the nozzle. However, it is quite possible that in the converging section, the relative accelerations of the two phases may be different so that slip is introduced there. The equations of motion (section 2.3) can therefore be analysed with $k_0 = 1.0$ and $n = 1.0$, and assuming arithmetic mean values of the slip ratio $\bar{k}$, [14], along the nozzle. The resulting values of the critical pressure ratio with k_t as a parameter, are plotted in figure 10, together with experimental data. The measured dimensionless choked flow rates are also compared with predictions from [31] in figure 11. On the whole, the agreement with the experimental data is better using the averaged slip ratio $\bar{k}$ than is obtained with the homogeneous theory (i.e. when $k_t = 1$). The theoretical mass flow rates and critical pressure ratios appear to follow the expected trend for $\delta_0 < 0.3$, i.e. that increasing slip leads to an increased flow rate and a corresponding increased pressure ratio.

The equations for the choking velocity c_L and the mass flow rate W, however, show a surprising contrast. If the slip ratio is increased from $k_t = 1.0$ the choking velocity predicted by [24] increases for $\delta_0 < 0.3$ but decreases for δ_0 greater than about 0.3; the experimental pressure ratios are also less than the corresponding values predicted by the homogeneous theory ($k_t = 1.0$) in the range $0.3 < \delta_0 < 0.8$. The decrease in the theoretical pressure ratio when $k_t > 1.0$ and $\delta_0 > 0.3$ is as observed with the experimental data in the range $0.3 < \delta_0 < 0.8$. However, the theoretical mass flow rate actually *increases* over the whole range of δ_0. This is due to the greater effect of k_t on the mixture density, from [2], than on the velocity in [24], the mass flow rate being the product of the two.

So far we have assumed that the foam expands isothermally, so that the polytropic expansion index n is equal to 1.0. In view of the complexity of the problem it seems impossible to separate slip effects from deviations caused by non-isothermal behaviour. The effect of non-isothermal gas expansion on the critical pressure ratio is, nevertheless, apparent from figure 10 for $\delta_0 > 0.8$ where the data shows better agreement with the equilibrium pressure

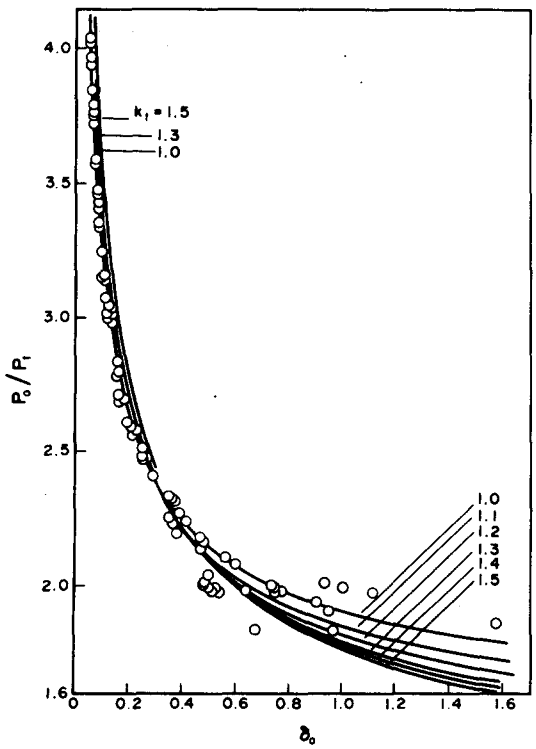

Figure 10. Effect of throat velocity ratio (k_t) on the theoretical critical pressure ratio: ——, theory [26]; ○, measurement; $k_0 = 1$; $\bar{k}$ according to [14]; $n = 1$; $dU_G/dU_L = 1$; $T_L = 25°C$.

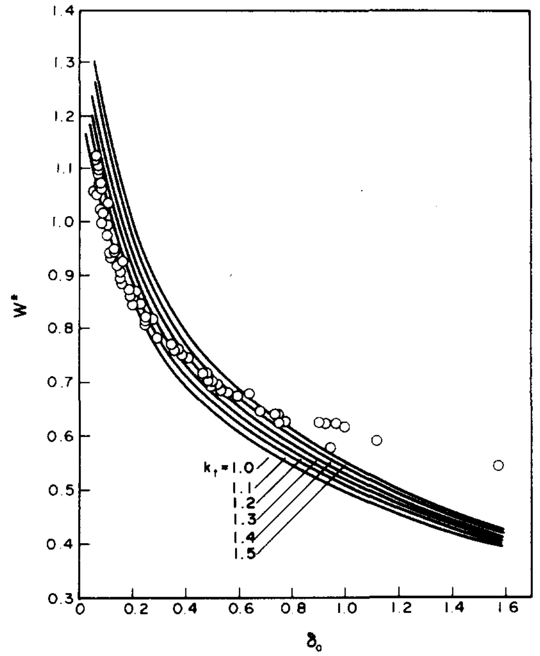

Figure 11. Effect of throat velocity ratio (k_t) on the theoretical dimensionless choked flow rate when gas expansion is isothermal, i.e. $n = 1$: ——, theory [31]; ○, measurement; $k_0 = 1$; $dU_G/dU_L = 1$; $T_L = 25°C$.

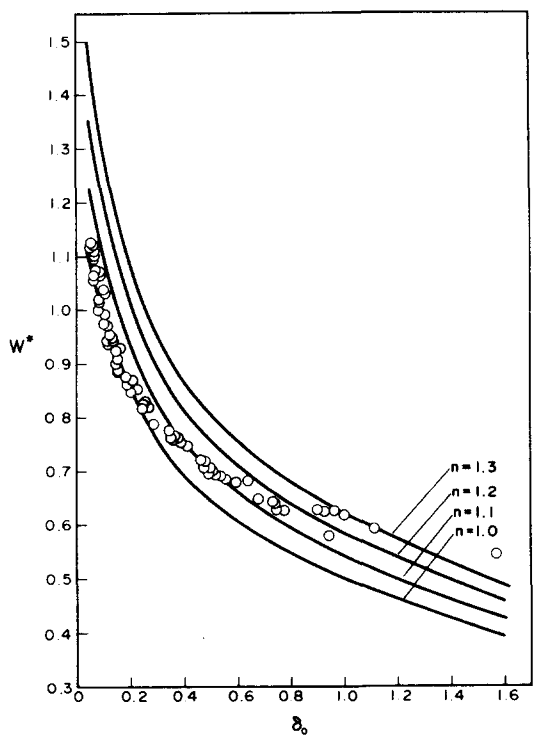

Figure 12. Effect on non-isothermal gas expansion ($n > 1$) on the theoretical dimensionless choked flow rate in the absence of relative velocity, i.e. $k = 1$: ——, theory [32]; ○, measurement; $dU_G/dU_L = 1$; $T_L = 25°C$.

ratios (assuming $k_t = 1$), than with $k_t > 1$, contrary to results at lower δ_0. It is, therefore, useful to allow n to vary and see if changes in the predicted pressure ratios and mass flow rates are in agreement with observation. Figure 12 shows how the dimensionless flow rate varies with δ_0 for various values of n. The throat velocity ratio is assumed to be 1.0. The relative acceleration term dU_G/dU_L has been taken as 1.0; when $dU_G/dU_L > 1$, the theoretical choked pressure ratios and flow rates are both increased. In figure 12, the experimental data lie mainly between the theoretical lines for $n = 1.0$ and $n = 1.3$. Furthermore, as δ_0 increases, there is a shift in the data from isothermal toward adiabatic behaviour. This is to be expected, because increasing δ_0, and hence the throat volume ratio δ_t, implies that the gas bubbles or slugs will become quite large in the throat and the heat associated with expansion of the gas will not be transferred to the liquid in the short time available.

Taken together, figures 11 and 12 imply that for low gas volume ratios, i.e. $\delta_0 < 0.3$ approx, the foam follows the homogeneous model for choked flow, and that the assumptions of isothermal expansion and no-relative velocity hold. In the range $0.3 < \delta_0 < 0.7$, the effect of relative motion between phases is more important than non-isothermal behaviour. Beyond $\delta_0 = 0.7$, both effects are significant.

6. CONCLUSIONS

Foam flow through a convergent–divergent nozzle (initial volume ratio, $0.053 < \delta_0 < 1.57$) produced by passing the two phases through a centrifugal pump has been investigated experimentally. The following remarks are appropriate.

(1) The foam flow initially containing bubbles of diameter $\sim 6 \times 10^{-5}$ m, is an ideal example of a homogeneous fluid. The measured mean throat velocities, choked flow rates and critical pressure ratios all indicate excellent agreement with the corresponding values predicted by the homogeneous frictionless nozzle flow theory for $\delta_t < 0.8$.

(2) For $0.8 < \delta_t < 2.98$, the foam flow can still be approximately described by the equilibrium theory although departures increase with increasing δ_t.

(3) The role of the detergent may be seen as aiding in the creation and maintenance of the system in equilibrium by decreasing bubble sizes as a result of reduced coalescence effects.

(4) Departures from the equilibrium theory are shown qualitatively to be due to the effect of relative motion becoming important (slip ratio $1 < k < 1.5$) in the range $0.3 < \delta_0 < 0.7$. The gas expansion may also be non-isothermal in this range but its effect appears to be much less than that due to velocity non-equilibrium. At higher values, i.e. $\delta_0 > 0.7$, non-isothermal effects may become increasingly significant, the polytropic gas expansion index (n) becoming greater than 1.0. The exact contribution of n to the critical pressure ratio is however, uncertain, due to slip also being present.

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